Let P be the point $(1,-4,-3)$. Let Q be the point $(-1,-2,5)$. **SCORE: _____ / 18 PTS**

Let R be the point such that $\overrightarrow{PR} = 2\overset{\rho}{j} - 2\overset{\rho}{k}$.

ALL ITEMS WORTH 1 POINT UNLESS OTHERWISE INDICATED

[a] Find the coordinates of R.

$\langle x-1, y-{}^{-}4, z-{}^{-}3\rangle = \langle 0,2,-2\rangle$

$x-1=0 \qquad x=1$

$y+4=2 \qquad y=-2$

$z+3=-2 \qquad z=-5$

$\underline{(1,-2,-5)}$

[b] Find the vector of magnitude 6 in the same direction as $\overrightarrow{PQ}$.

$6\cdot\frac{1}{\|\overrightarrow{PQ}\|}\overrightarrow{PQ}$

$= \underline{6}\cdot\underline{\frac{1}{6\sqrt{2}}\langle -2,2,8\rangle}$

$= \underline{\langle -\sqrt{2}, \sqrt{2}, 4\sqrt{2}\rangle}$

$\overrightarrow{PQ} = \langle -1-1, -2-{}^{-}4, 5-{}^{-}3\rangle$

$= \underline{\langle -2,2,8\rangle}$

$\|\overrightarrow{PQ}\| = \sqrt{4+4+64}$

$= \sqrt{72} = \underline{6\sqrt{2}}$

[c] Find $\overrightarrow{PQ} \times \langle -2,-1,3\rangle$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 8 \\ -2 & -1 & 3 \end{vmatrix}\begin{matrix} \vec{i} & \vec{j} \\ -2 & 2 \\ -2 & -1 \end{matrix} = 6\vec{i} - 16\vec{j} + 2\vec{k} - (-8\vec{i} - 6\vec{j} - 4\vec{k})$$

$= \underline{\langle 14,-10,6\rangle}$

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SANITY CHECK:

$\langle 14,-10,6\rangle\cdot\langle -2,2,8\rangle = -28-20+48 = 0$ ✓

$\langle 14,-10,6\rangle\cdot\langle 2,-1,3\rangle = -28+10+18 = 0$ ✓

THIS WILL BE WORTH POINTS ON UPCOMING TESTS

[d] Find the equation of the sphere with P and Q as endpoints of a diameter.

CENTER $= \left(\frac{1+-1}{2}, \frac{-4+-2}{2}, \frac{-3+5}{2}\right) = \underline{(0,-3,1)}$

RADIUS $= \frac{1}{2}\|\overrightarrow{PQ}\| = \underline{3\sqrt{2}}$

$\underline{x^2+(y+3)^2+(z-1)^2 = 18}$

[e] Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (Your answer should be in radians.)

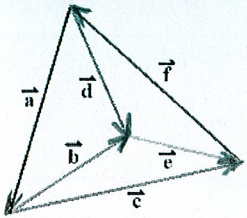
2

$\cos^{-1}\frac{\langle -2,2,8\rangle\cdot\langle 0,2,-2\rangle}{\|\langle -2,2,8\rangle\|\,\|\langle 0,2,-2\rangle\|} = \cos^{-1}\frac{0+4-16}{\underline{6\sqrt{2}}\cdot\underline{2\sqrt{2}}} = \cos^{-1}\frac{-12}{24} = \underline{\cos^{-1}\frac{-1}{2}}$

$= \underline{\frac{2\pi}{3}}$

Write vector $\vec{e}$ in terms of vectors $\vec{a}$, $\vec{d}$ and/or $\vec{f}$ in the diagram below.

SCORE: _____ / 2 PTS



$$\vec{d} + \vec{e} = -f$$

$$\vec{e} = -\vec{d} - \vec{f}$$

2

Consider the sphere with equation $x^2 + y^2 + z^2 + 12x - 8y + 2z + 5 = 0$. **SCORE: _____ / 7 PTS**

[a] Find the equation of the xz – trace, and describe that trace clearly and briefly.

$\hookrightarrow y = 0$

1 $x^2+12x+36+y^2-8y+16+z^2+2z+1 = -5+36+16+1$ 1

1 $(x+6)^2+(y-4)^2+(z+1)^2 = 48$ $\frac{1}{2}$

$(x+6)^2+(0-4)^2+(z+1)^2 = 48$

1 $(x+6)^2+(z+1)^2 = 32 \rightarrow$ CIRCLE CENTER $(-6, 0, -1)$ $\frac{1}{2}$

RADIUS $4\sqrt{2}$ $\frac{1}{2}$

[b] If you start at the center of the sphere, then move 3 units down and 2 units to the right, find the coordinates of where you are now.

$(-6, 4+2, -1-3) = (-6, 6, -4)$ $1\frac{1}{2}$

Find all octants in which $xyz > 0$.

SCORE: _____ / 3 PTS

$x>0,\ y>0,\ z>0 \longrightarrow O_1$

$x>0,\ y<0,\ z<0 \longrightarrow O_{4+4} = O_8$

$x<0,\ y<0,\ z>0 \longrightarrow O_3$

$x<0,\ y>0,\ z<0 \longrightarrow O_{2+4} = O_6$

OCTANTS $\underbrace{1,}_{1}\ \underbrace{3,}_{1}\ \underbrace{6,}_{1}\ \underbrace{8}_{1}$